THE CHINESE UNIVERSITY OF HONG KONG
DEPARTMENT OF MATHEMATICS

MMAT5220 Complex Analysis and its Applications 2016-2017
Suggested Solution to Assignment 3

1 If z_0 does not lie inside C, by Cauchy-Goursat theorem we have

$$\int_C \frac{dz}{z-z_0} = 0$$

Otherwise, by Cauchy integral formula, we have

$$\int_C \frac{dz}{z-z_0} = 2\pi i(1) = 2\pi i$$

2 (a) Since the function $f(z) = \dfrac{1}{z^2-1}$ is analytic inside the contour C_1 except the point $z=-1$, by Cauchy integral formula, we have

$$\int_{C_1} \frac{1}{z^2-1}dz = \int_{C_1} \frac{1/(z-1)}{z+1}dz = 2\pi i \frac{1}{(-1)-1} = -\pi i$$

(b) Denote $C(z,r)$ be the circle centered at z with radius r in counter-clockwise direction. Then we have

$$\int_{C_2} \frac{3z-2}{z^2-z}dz = \int_{C(0,\frac{1}{2})} \frac{3z-2}{z^2-z}dz + \int_{C(1,\frac{1}{2})} \frac{3z-2}{z^2-z}dz$$

By Cauchy integral formula, we have

$$\int_{C(0,\frac{1}{2})} \frac{3z-2}{z^2-z}dz = \int_{C(0,\frac{1}{2})} \frac{(3z-2)/(z-1)}{z}dz = 2\pi i \frac{3(0)-2}{0-1} = 4\pi i$$

and

$$\int_{C(1,\frac{1}{2})} \frac{3z-2}{z^2-z}dz = \int_{C(1,\frac{1}{2})} \frac{(3z-2)/z}{z-1}dz = 2\pi i \frac{3(1)-2}{1} = 2\pi i$$

As a result, we have

$$\int_{C_2} \frac{3z-2}{z^2-z}dz = 4\pi i + 2\pi i = 6\pi i$$

(c) Decomposite C_3 into T_1 and T_2, where T_1 and T_2 are the left and right parts of C_3 respectively. Then we have

$$\int_{C_3} \frac{2z^2-z+1}{(z-1)^2(z+1)}dz = \int_{T_1} \frac{2z^2-z+1}{(z-1)^2(z+1)}dz + \int_{T_2} \frac{2z^2-z+1}{(z-1)^2(z+1)}dz$$

By Cauchy integral formula, we have

$$\int_{T_1} \frac{2z^2-z+1}{(z-1)^2(z+1)}dz = \int_{T_1} \frac{(2z^2-z+1)/(z-1)^2}{z+1}dz = 2\pi i \frac{2(-1)^2-(-1)+1}{((-1)-1)^2} = 2\pi i$$

and

$$\int_{T_2} \frac{2z^2-z+1}{(z-1)^2(z+1)}dz = -\int_{-T_2} \frac{(2z^2-z+1)/(z+1)}{(z-1)^2}dz = -2\pi i \left(\frac{2z^2-z+1}{z+1}\right)'|_{z=1} = -2\pi i$$

As a result, we have

$$\int_{C_3} \frac{2z^2-z+1}{(z-1)^2(z+1)}dz = 2\pi i - 2\pi i = 0$$